# GT zusammenfassung

## ① Überblick, Beispiele, anwendungen

Normalform (matrix)

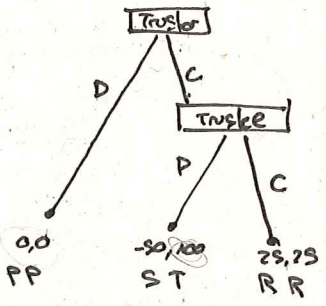
**Nullsummenspiel** elf meter

| | | L | R |
|---|---|---|---|
| | L | +1 | -1 |
| potter | R | -1 | +1 |

**Vertrauensspiel**

→ (extensivform) entscheidungsbaum

↳ dynamic




"Rückwärtsinduktion" Truster is titfordat, therefore D. Nobody wins

T > R > P > S

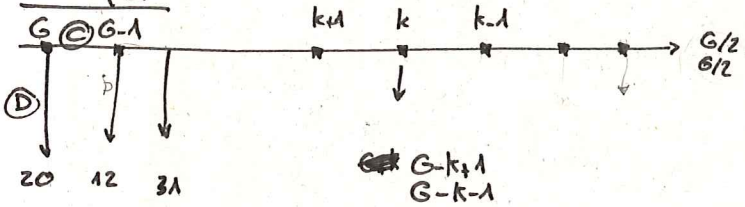
differenz (T-R) (75)

0 0 25 25 25 25

Mechanismus for best outcome (design)

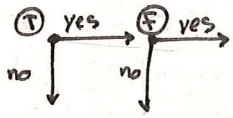
lösung: kaution, deposito (75)

**Münzspiel**




Sozale dilemma

**Salomos urteil**




(-s, W_f - s)

0 < s < W_f < S < W_t

"far division

→ if they both say "yes", we give it to the 2nd

**Golden BALLS**

| p1 \ p2 | C | D |
|---|---|---|
| C | 50,50 | 0, 1k |
| D | 1k, 0 | 0,0 |

PD

| | | |
|---|---|---|
| 3 3 | 1 4 |
| 4 1 | 2 2 |

## ② Normalform spiele (koordination, gemischte motive und nash-gleichgewicht)

**Koordinationspiel** (choose number)

(+probleme, standardisierung)

| p1 \ p2 | L | R |
|---|---|---|
| L | 11 | 00 |
| R | 00 | 11 |

normalform: • n players
• strategies for each
• 00 winning function

- n players i = 1, 2, ..., n
- $s_i$ is strategy for player i (eg. L)
- $S_i$ is set of strategies for player i (eg. {L, R})
- $s = (s_1, s_2, \dots, s_n) \mid s_i \in S_i$ is strategy profile
- $S = S_1 \times S_2 \times \dots \times S_n$ = set of all strategies
- $u: S \to \mathbb{R}^n$ payoff for each player $u_i(s)$

eg.

$s_{-i}$ : strategy profile without player i

## Nash-gleichgewicht

bestantwort: for player i, knowing the choice of other players, no change in strategy will give him better payoff

$s^*$ (NG)

↳ $s \in S / \forall i \in \{1,\dots,n\}$ $s_i$ is beste antwort
(nessuno ha la stimolo di cambiare)

$S^* = (s_1^*, s_2^*, \dots, s_n^*)$ is NG $\Leftrightarrow$

$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$ → also =

$>$ (strikt)

(1) cambio pareto inferior

(not always just 1 solution)

**Dominant strategy:** $s_i^*$ is $\Leftrightarrow u_i(s_i^*, s_{-i}) \geq (s_i, s_{-i})$

strikt → (>)

(eliminate dominated lines → no use in them)
↳ it yields NG

## Maximin

| | $p_2$ | | | | min row |
|---|---|---|---|---|---|
| $p_1$ | 4 2 | 3 0 | 7 -1 | -12 4 | -12 |
| | 3 1 | 0 1 | 1 1 | 6 2 (NG) | 0 |
| | 2 0 | 2 1 | 6 2 | 5 3 | (2) max of min |
| | 0 | 0 | -1 | (2) | |

(best in worst case scenario)
sicherheitsstrategie, WCS (worst case strategy)

$s_i^*$ is $\Leftrightarrow \min u_i(s_i^*, s_i) \geq \min(s_i, s_{-i})$

## Pareto OPTIMUM

pareto inferior: at least one player can improve its payoff without damaging others (others can also improve it) → pari overdose in diagonale

inefficient

optimum: not pareto inferior (not exclusive)
efficient

| | | | |
|---|---|---|---|
| 4 2 | 3 0 | 7 1 | 0 0 |
| 3 1 | 0 1 | 1 1 | 5 2 |
| 0 0 | 2 1 | 6 2 | 5 3 |

## Beautiful mind game

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| bionde | T | T/2 | T/3 | T/4 |
| brune | R | R | R | R |

← how many others talk to the blonde

## Semplici motivi

→ strict order of preferences

$(4.3.2.1)^2 = 576$ → 72 (78) games

2x2

| | interesse |
|---|---|
| • koordinationsspiele | gemeinsame |
| • gemischte motive | teils gemeinsame / konfliktierende |
| - battle of sex | |
| - assurance | |
| - prisoner | |
| - chicken | |
| • nullsummenspiele | antagonistische |

**BATTLE OF SEX**

| ♂ \ ♀ | kino | fuss |
|---|---|---|
| kino | 4 3 | 2 2 |
| fuss | 1 1 | 3, 4 |

**ASSURANCE GAME** (stag hunt)

| hunter 1 \ hunter 2 | deer | bunny |
|---|---|---|
| deer | 3 3 | 1 2 |
| bunny | 2 1 | 2 2 |

| | | |
|---|---|---|
| 4 4 | 1 3 | |
| 3 1 | 2 2 | |

**GEFANGENENDILEMMA**
(illegale transaction anonym)
PD prisoners dilemma

| | C | D |
|---|---|---|
| C | 3 3 | 1 4 |
| D | 4 1 | 2 2 |

→ better for both (pareto opt)

$T > R > P > S$
↳ $R > (T+S)/2$

temptation, reward, punishment, sucker's payoff

but DD is
- NG
- maximin
- dominante

## ③ SEQUENTIELLE SPIELE
## GLAUBWÜRDIGKEIT VON DROHUNGEN, TEILSPIELPERFEKTHEIT

**chicken**
(small change in PD)

| | C | D |
|---|---|---|
| C | 3 3 | 2 4 |
| D | 4 2 | 1 1 |

no dom

crash

Kahn → throw the steering wheel out the window

CG, SG (snowdrift)

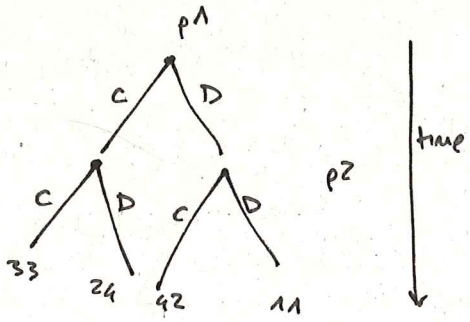
solves this problem

→ 3rd NG in mixed strategy

## SPIEL IN EXTENSIVE FORM

- node ≡ player
- edge ≡ decision

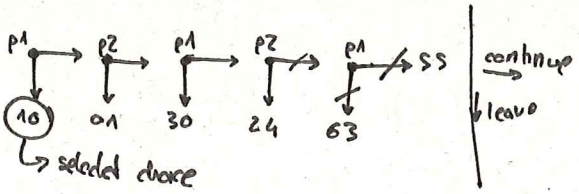
simultaneous chicken




common knowledge
CKR ( « rationality)

## THE BOTTLE IMP

CENTIPEDE GAME




Rückwärtsinduktion

## ④ Nutzenmessung: GESCHICHTE, PARADOXIEN, ANOMALIEN

chevalier de méré game

Value of the game: $\Pi(A) = p(A) \cdot x$

x: gewinn
P(A): probability A wins

st. petersburg paradox

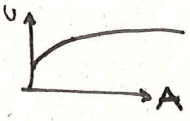
win $2^n$ € for sequence of n heads
game ends with tail

$$\Pi = \sum_{i=0}^{\infty} p_i A_i \quad \text{(payment)} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots \to \infty$$

Bernoulli: • petersburg sono finiti

$$U = a \cdot \log(A + b)$$

$$\frac{dU}{dA} > 0 \qquad \frac{d^2U}{dA^2} < 0$$




Neumann-Morgenstern (profitto intervallscala)

lotteria L: $a_1 = 1k$ $p_1 = 0{,}3$; $a_2 = 2k$ $p_2 = 0{,}5$; $a_3 = 3k$ $p_3 = 0{,}2$ $\quad \Sigma p_i = 1$

$u(a_i)$ = → psychologische nutzen of real world unit SUBJECTIVE

$L(a_2) \sim L[(a_1, 1-p), (a_3, p)]$ (p=1)
↳ indifference between lotteries

$u(a_2) = p \cdot u(a_1) + (1-p) \cdot u(a_3)$

nutzenfunktion

SEU-theory (subjektiv erwarteter Nutzen) expected ability profitto
Savage, Allais Paradox

| | | 1-10 | 11 | 12-99 | 100 | E[x] |
|---|---|---|---|---|---|---|
| choose | $A_1$ | 1 k | 1 k | 1 k | 1 k | 1k |
| | $B_1$ | 5 k | 1 k | 1 k | 0 | 1,39 k ← better |
| | $A_2$ | 1 k | 1 k | 0 | 0 | 0,11 k |
| | $B_2$ | 5 k | 0 | 0 | 0 | 0,5 k ← |

## TEILSPIELPERFEKTHEIT

→ refinement of NG

(subgame perfect equilibrium) SPE

teilspiel H of spiel G (extensive) which is itself a game

NG is teilspielperfect if it's NG of all subgames (which can be reached)

▽ ▼ not perfect

## GLAUBWÜRDIGKEIT VON DROHUNGEN

"I'm gonna select D, so you better go with C"
p2: →

| p1 \ | CC | CD | DC | DD |
|---|---|---|---|---|
| C | 3 3 | 3 3 | 2 4 | 2 4 |
| D | 4 2 | 1 1 | 4 2 | 1 1 |

(teilspiel)

CC: reply C for C and C for D

p1 advantage: can estorcere his advantage

glaubwürdig = credible ⇔ teilspielperfekt

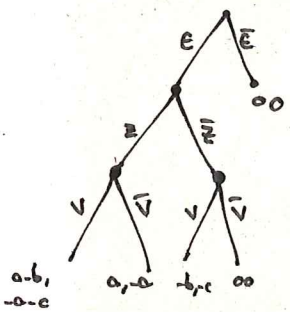
## ULTIMATUMGAME

- uno taglia
- l'altro – accetta
  \ rifiuta (nessuno prende)

e.g. divisione 100.-

every subdivision is ~~finite~~, and PO
~~perfect~~ NG: p1 offre quantità minima che p2 accetta
↳ also p1 offre 0, p2 accetta

## RATIONAL RANSOM




E = make ransom
Z = pay
V = giveaway info

a = $1M
b = cash in due
c = penalty for fisco

Perfect solution: Ē !

t<a.

(start, within)

How to become millionaire

→ next president ♀ result

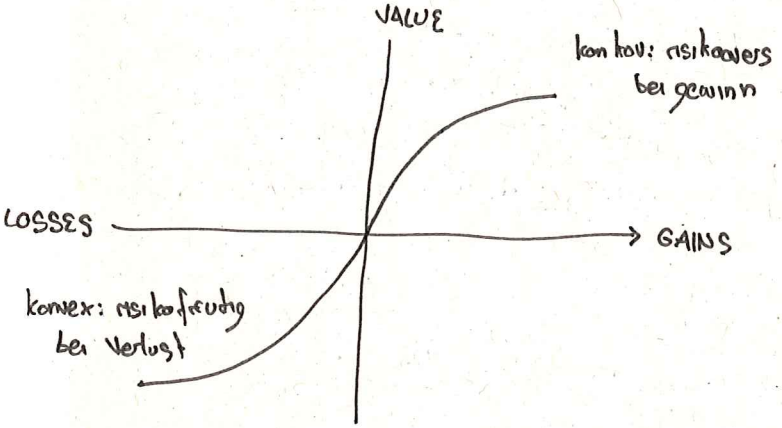
| you say \ result | M | F |
|---|---|---|
| M | +2 | -1 |
| F | -1 | +2 |
| | +1 | +1 |

bet M:F . 1:2
M:F - 2:1

⇨ How you frame it (paradox) (Kahnemann, Tversky)

WIN-frame → Risiko averse
LOSE-frame → Risiko freudige (gioiosa)

Logik ≠ Psychologik




## SOCIAL CHOICE theory

- individual preferences → collective choice (democratic)
- rules for votation

CONDORCET-PARADOX

| | p1 | p2 | p3 |
|---|---|---|---|
| A | 3 | 2 | 1 |
| B | 2 | 1 | 3 |
| C | 1 | 3 | 2 |

choice (?)

⚠ cyclic preferences

⚠ regole di votazione influenzano risultato

BORDA RULE / COUNT (give to each option a score)

→ it elects broadly acceptable options (rather than those who have a majority)

(CONSENSUS BASED option)

| flag over vom | p1 | p2 | p3 | p4 | p5 | |
|---|---|---|---|---|---|---|
| | 2 | 2 | 1 | 1 | 0 | → 6 |
| | 1 | 1 | 0 | 0 | 2 | 4 |
| | 0 | 0 | 2 | 2 | 1 | 5 |

but, if no chocolate

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 2 |
| - | - | - | - | - | |
| 0 | 0 | 1 | 1 | 1 | 3 |

result
→ depends on unimportant (wrong) alternatives

Arrow theorem (impossibility) → no ranked order voting system can convert ranked preferences of individuals into a community-wide ranking while satisfying those criteria. (they are incompatible)

4 axioms
- unrestricted domain
- pareto efficiency
- independence of irrelevant alternatives
- no dictatorship

→ with 2 alternatives the majority rule satisfies all 4

CONDORCET-WINNER

pairwise abstimmung → if you win against both, then you are absolute winner (no cycles)

## ⑤ NULLSUMMEN SPIELE

→ chance games with antagonistic interests (society games)

konstantsummenspiel → has natürlich nullpunkt

| | |
|---|---|
| 4-4 | 3-3 |
| 1-1 | 2-2 |

bimatrix game

normed (lin. transf)

dominant strategy row

| | |
|---|---|
| 4 | 3 |
| 1 | 2 |

matrix game

SATTELPUNKT (row min, col max)

(with dominant strategy there is always sattelpunkt)

→ all sattelpoints have same value (in contrast to non-nullsum games)

SATTLEpoints can be interchanged

| R \ C | | | | |
|---|---|---|---|---|
| | 2 | 4 | 9 | 3 |
| | 6 | 5 | 7 | 5 |
| | 7 | 5 | 6 | 5 |
| | 2 | 4 | 7 | 4 |
| | 8 | 3 | 1 | 2 |

BISMARKSEE-BOMBARDIERUNG

| you \ S | | |
|---|---|---|
| | 2 | 1 |
| | 1 | 3 |

why these values??

↳ weak dominant strategy

# GEMISCHTE STRATEGIE

(no sattlepunkt oder dominante) (5)

| | $q_1$ | $q_2$ |
|---|---|---|
| $p_1$ | 2 | 3 |
| $p_2$ | 4 | 1 |

(2) R — C $(1-q_1)$

Erwartungswert:

$p_1q_1 \cdot 2 + p_1q_2 \cdot 3 + p_2q_1 \cdot 4 + p_2q_2 \cdot 1 = E$

payoff: $(E, -E)$

solution $z^*, s^*$ → they have to be indifferent for the other player

row: $2p_1 + 4(1-p_1) = 3p_1 + 1(1-p_1) \rightarrow p_1^* = \frac{3}{4} = 0{,}75$

col: $2q_1 + 3(1-q_1) = 4q_1 + 1(1-q_1) \rightarrow q_1^* = 0{,}5$

$-2p + 4 = 2p_1 1 \quad | \quad -1q + 3 = 3q + 1$

$3 = 4p \rightarrow \quad 2 = 4q \rightarrow$

$z^* = (0{,}75; 0{,}75)$

$s^* = (0{,}5; 0{,}5)$ → $E = \ldots = 2{,}5$

If you deviate from the strategy, you don't win neither lose.

Torwart, elfmeterschütze

| | |
|---|---|
| 1 | -1 |
| -1 | 1 |

$p = 0{,}5$
$q = 0{,}5$

Serve and return (TENNIS)
→ hard to take advantage (although not "random" like coin)

Allgemein

| | $q$ | $1-q$ |
|---|---|---|
| $p$ | a | b |
| $1-p$ | c | d |

$$p^* = \frac{d-c}{(a+d)-(b+c)}$$

$$q^* = \frac{d-b}{(a+d)-(b+c)}$$

Ableitungsmethode:

$E_2 = pq \cdot a + p(1-q)b + (1-p)qc + (1-p)(1-q)d$

$\frac{\partial E_2}{\partial p} \overset{!}{=} 0 \rightarrow q^*$ $\quad E_S = -E_2$

$\frac{\partial E_S}{\partial q} \overset{!}{=} 0 \rightarrow p^*$

| | R | P | S |
|---|---|---|---|
| R | 0 | -1 | 1 |
| P | 1 | 0 | -1 |
| S | -1 | 1 | 0 |

→ gemischte strat $= \frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

$E = 0$

CHICKEN GEMISCHTE STRATEGY

row: $3p + 2(1-p) = 4p + 1(1-p)$

$p+2 = 3p+1$

$p^* = q^* = \frac{1}{2} = 2p \rightarrow 0{,}5$

$E = 2{,}5 \quad \left(\frac{2 \cdot (1+2+3+4)}{8}\right)$

| | | p2: C $q$ | p2: D $1-q$ |
|---|---|---|---|
| p1: C | $p$ | 3 / 3 | 2 / 4 |
| p1: D | $1-p$ | 4 / 2 | 1 / 1 |

(symmetric but not PO)

no zero sumgame!

TIC TAC TOE

if this, I respond with that

2...9 1 3...9

# strategies for 2nd player? (first move) (3)

9.8?

$0.98 \ ?$ $\quad 8^9$

THEORETIS

1. ZERMELO → each zerosum game with finite #strategies and perfect info has $\geq 1$ sattlepoint

2. NEUMANN (minimax) → each zerosum game with _____ has "gleichgewicht" in pure or mixed strategies

3. NASH → every finite game (finite # players & strategies) has $\geq 1$ gleichgewicht in pure or mixed sbt

## (6) KARTELLE UND AUKTIONEN

Cournots duopol (due monopoli)

$P(q_1+q_2)$ // price
$C_i(q_i)$ // cost

$\Pi_i = P(q_1+q_2) \cdot q_i - C_i(q_i)$ // profit

→ find $q_i$ that maximizes $\Pi_i$ given $q_j$ $\quad \frac{\partial \Pi_i}{\partial q_i} \overset{!}{=} 0$

es

richiesta $P = 100 - (q_1 + q_2)$ // price

$q_i$: // produced quantity
(assume no costs)

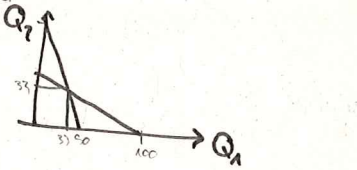
$E_1 = P q_1 = q_1(100 - q_2) - q_1^2$ // ricavo

$E_2 = P q_2 = q_2(100 - q_1) - q_2^2$

$q_1 = 50 - 0{,}5 q_2$
$q_2 = 50 - 0{,}5 q_1$

$q_1^* = q_2^* = 33{,}\overline{3}$

→ $P = 33{,}\overline{3}$
$E = PQ = \ldots$ (another test to prove NG)




MONOPOL

$Q = q_1 + q_2$ to maximize combined gain
$P = 100 - Q$
$E = PQ = 100Q - Q^2$
$\frac{\partial E}{\partial Q} = 100 - 2Q \overset{!}{=} 0 \rightarrow Q = 50$ split 25 each
$P_M = 100 - 50 = 50$
$E_M = 50 \cdot 25 = 1250 > \ldots$

profitto a scapito dei consumatori

it's not NG! each would profit more not to cooperate

| | kartel | duopol |
|---|---|---|
| kartel | 1250, 1250 | 1042, 1389 |
| duopol | 1389, 1042 | ..., ... |

GD

kartellen . sfruttano cluent
• sono instabili (ancora di più se non si è trasparenti in quantità prodotta /scorte/...

⇨ KRONZEUGEN REGELUNG
(the first to talk isn't punished) or pays less
(penalty)

## AUKTIONEN

result is interdependent
there are different types - rules can influence outcome
auktiondesign
• open auction
english ↗
hollandisch ↘
• hidden auction
- erstpreisauktion (1st win/pay)
- zweitpreisauktion
(1st win, $ pays 2nd

CVA common value a.
obj value, unknown exactly

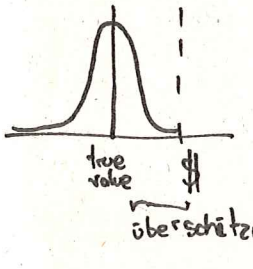
PVA private value
subj value

other rules:
• 1st win/pays, 2nd also pays
• 1st —, all other also pay
• verdeckt (secret) • open

VOX POPULI → show peso uomo → nell'insieme nulla preciso!

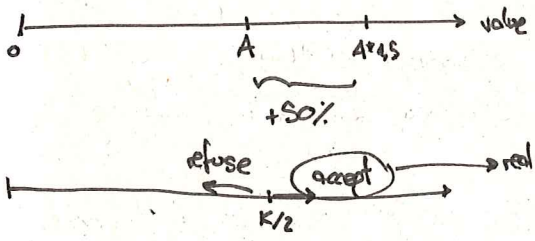
DER FLUCH DES GEWINNENS
chi sopravvaluta il value (subj) → vince
→ ma svantaggiosa

"la maledizione del vincitore"




TAKING OVER A COMPANY

estimated: $[0, k]$, average $\frac{k}{2}$




$[0, \frac{k}{2}]$
average $\frac{k}{4}$
$+50\% = \frac{3}{8}k < \frac{4}{8}k$
payed

wallet auction:

NG PO strategy: bid until twice your $ can rent

$x_1, x_2 \rightarrow$ $ in wallet
$s^* = (\text{bid } 2x_1 \text{ (max)}, \text{bid } 2x_2 \text{ (max)})$
$v(s^*) = (u_1 \geq 0, u_2 \geq 0)$
q = guadagno

| $x_1$ ? $x_2$ | $u_1$ | $u_2$ |
|---|---|---|
| > | $v_1+v_2-2x_2$ | 0 |
| = | 0 | 0 |
| < | 0 | $v_2-v_1>0$ |

$v_1 \; v_2 \; \dots \; v_{n-1} \; v_n$

each bids $b \leq v_{max}$ (personal)
$v_n = v_{n-1} + \Delta$ (min increase) → if small, 2nd best price is paid

Zürich auction unobtainable
bid $b \leq (v^1) < v$
↳ value indebatable shmarda

WILLIAM VICKREYS 2nd price auction
(pay 2nd) → siero della vendita

| | case 1 | case 2 | case 3 |
|---|---|---|---|
| $b > v$ | r v b | v r b | v b r |
| ? $<$ | r b v | b r v | b v r |
| $=$ | r v = b | r v = b | b = v r |

v = value to p1
b = bid of $p_1$
r = $p_2$

gain:

| strategy (3) | | | |
|---|---|---|---|
| v-r > 0 | v-r < 0 | | 0 |
| | 0 | | 0 |
| | v-r > 0 | | 0 |

NG is strategy 3 → they each bid real value
(schwach dominant strategie)

RET (revenue equivalence theorem)
pay same price for all actions of same type

# 7. SOCIAL DILEMMAS

## Höchste zahl wins (LARGE NUMBERS)

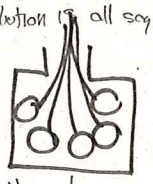
→ write # on pper
→ higher wins
→ payoff 500.-/number

→ instable, not robust system
→ N-1 coop, but 1 "bad apple" enough to ruin it
~~NG~~ → solution is all say 1 (PO, symm)

## Minte experiment

theory NPD
rushing strategy (D) is dominant
even if other all planned in advance

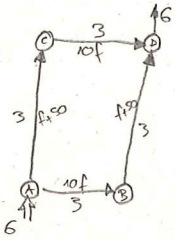
if incentives are required, it's a matter of coordination (often ethical first come - first serve)



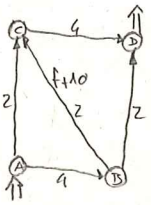
N-people generalization of "Battle of sex"

## BRASS paradox

variant of PD




each 83 min




each 92 min
although street has been widened

## SOCIAL DILEMMAS

→ the result of interaction isn't optimal PO (not efficient)
→ could have better outcome if all N player made a contract

→ **collective good** (environment, ...)
public good
⚠ non-exclusion principle

→ **coordination problems** (stretbie, point situation)
→ **social exchange**

## RIVALITY IN CONSUME

| | EXCLUSION Y | EXCLUSION N |
|---|---|---|
| Y | private good (mensa) | allmendegut (pesci) |
| N | Clubgut (park/golf) | public good often clubgut but (environment) |

people

## X or Y

select x or y
price $A_x$: 2. #x
$A_y$: 3. #x +3

es N=10, #x=5, $A_x$=10, $A_y$=15

## N-people PD (N-GD)

es 100 players

| | 0 | 1 | 2 | 3 | ... | 98 | 99 |
|---|---|---|---|---|---|---|---|
| C=X | 2 | 4 | 6 | 8 | ... | 198 | 200 |
| D=Y | 3 | 6 | 9 | 12 | ... | 297 | 300 |

← other x players
dominant

[D is NG, but is PI, not PO
C is PO, but not NG (everyone has to cooperate)

[D(x) > C(x+1)
D(0) < C(N) → social dilemma

WIKI CAMPAIGN
ORGANSPENDEN (collective good)
C= donorcard sforzo k
D= no " 0
solution: have to explicitly say no (default = yes)

## PUBLIC GOOD EXPERIMENT (game, PGG)

→ get $5
→ invest part in collective fond ($x_i$)
→ doubled & distributed evenly

| | p1 | p2 | p3 | |
|---|---|---|---|---|
| invest | 0 | 1 | 5 | Σ=6 |
| fund | 4 | 4 | 4 | +12/3 |
| | 9 | 8 | 4 | |

$U_i$ → max when $x_i$=0 dominant NG
(strategia dell'opportunista (trittbrett)

↳ the more collaborative bst

### other example

4 player, start $20
invest part
each receives 0,4 of founds (+60%)
dominant & NG: profitabre (D) → homo economicus

### sanktionmöglichkeit

→ give 0-10 sanktion point. each giver per point pays $1, receiver $3

NG: full cooperation & sanction of non-cooperative
teilspielperfect NG: profitatore (D) → homo economicus: nobody pays in fonds, because no sanction will happen. (nicht glaubwürdig drohung)

## Experiment design

2×6 rounds PGG → aftereach round groups built again (stranger matching → no wiederholte game)
one/nit sanktion, don't know other players

## Altruistic punishment in humans

→ you pay for punishment, but later you profit from their cooperation.
→ why we sanction although it costs? (not homo economicus)
⇒ not only material payoff → fairness! (reciprocity)

## Climate change (Milinski experiment)

start $20. 10 rounds, 6 player. give 0,2,4$. must reach total 120$
if not, lose %: 90/50/10% based on risk
far sharer + altruistic (same) free rider logical
→ they invest only with high risk of loss (sometimes they are profitatori)

collective rationality → PO, efficiency (invisible hand)
social dilemma, no rational → PI (invisible fist)

## How to REACH COOPERATION IN SOCIALE DILEMMAS

→ sanctions (no coop non é sanzionabile se non fatta)
→ wiederholte interaktion
→ social norms → reputation (ebay)

abzeigen [+§-] • kooperativ/fair/effizient/einfach

**Sfruttamento pesci:** ogni giorno pescherecci rientrano location

# 6. VOLUNTEER'S DILEMMA

es. accensione impianto

VOD: min 1 has to coop (cost K) to perform collective GOOD (U) $U > U-K > 0$

es. riordo

2 people see it and do nothing → non ognuno assume frazionato
→ riceve sola indifferenza

⇒ **hypothesis of responsability distribution**
(bystander intervention example)

→ the less people there are, the more they intervene & in less time

**VOD** collective good game → 1 coop is enough

N players

| | 0 | 1 | 2 | ... | N-1 |
|---|---|---|---|---|---|
| C | U-K | U-K | U-K | ... | U-K |
| D | 0 | U | U | ... | U |

← others coop players

save

2-VOD

| | C | D |
|---|---|---|
| C | U-K | U-K |
| D | U | 0 |

→ no dominant
→ N asymmetric NG: $s = (C, D, D, \dots D)$
→ Gleichgewicht in mix-strategy

**Rational choice**
→ coop as moral obligation
→ maximin: coop, payoff U-K → problem: if I know better choice would be D

**Gemischte strat:**

$$E_i = q_i U\left(1 - \prod_{j \neq i}^{N} q_j\right) + (1-q_i)(U-K)$$

$$\frac{\partial E_i}{\partial q_i} = K - U\prod_{j \neq i}^{n} q_i \stackrel{!}{=} 0 \quad \rightarrow \text{symmetric solution}$$

$$q_0 = \left(\frac{K}{U}\right)^{1/(n-1)}$$

why $p^* = 1 - q^*$ ?

**Indifferenz pinzip**
→ select $q = 1-p$ such that others are indifferent in taking C or D

$$p = 1 - \sqrt[N-1]{\frac{K}{U}}$$

prob of C: decreases as N ↑

$p \equiv C$
$q \equiv D$

→ **Victim paradox**
→ should wish for few people watching
"I feel less responsible", "maybe others know more" ↳ paradox

individual plane ↖ collective plane ↙

$P(\text{nobody cooperates}) = q^N$
$P(\text{success}) = 1 - q^N = 1 - \left(\frac{K}{U}\right)^{\frac{N}{N-1}}$

⇒ If others don't think it's a problem, probably it isn't.
↳ if you know about it, it goes better

**possible examples**
invest R&D, wait concurrence
sanctioning if costly
decentralized computer networks

→ save classified as PD, others as CG
es emails: response ↘ as simultaneous send ↗

**Asymmetric VOD**
$U_i, K_i$ for $i = 1 \dots N$, $U_i > U_i - K_i > 0$
→ eg. one strong player, rest weak
$U_s - K_s > U - K$
⇒ asymmetric strategy: $s = (C_s, D, D, \dots)$ rational choice

paradox: sfortunato player forte
→ ha la minor possibilità di giocare

$$p_i^* = 1 - \frac{U_i}{K_i}\left(\prod_{j=1}^{N} \frac{K_j}{U_j}\right)^{\frac{1}{N-1}}$$

**Sanctioning dilemma**
who punishes norm violator?
↳ depending on penalty

eg. chimpanzees in asymmetric collective action problem
→ higher in rank do it (egoistic gene)

# 8. EVOLUTION OF COOPERATION

Coord. probl → # of kisses (2/3)

**GD**

| | C | D |
|---|---|---|
| C | 3 3 | 1 4 |
| D | 4 1 | 2 2 |

central authority → cooperating
solitario, trouble, conta
society | nature

State of nature

**Hobbes**
Leviathan: reduction of T through punishment

**Wiederholte spiel**
→ is there "inner/erdogen" solution, or spontaneous order?
interaction: infinite/unknown # of games

**Supergame** 2-GD is base game

- **unbedingt strat** (incondizionata)

D: DDDD
C: CCCC
and: DDCD...

- **bedingt strat** (friedman)
start by cooperating until other defects → then always D
→ eterna ritorsione (trigger-strategy)

**Axelrod** computer tornamento

5 strat: 1 CI programmate + 1 random
round robin: play against self & others → 200 turns

| | C | D |
|---|---|---|
| C | 3 3 | 0 5 |
| D | 5 0 | 1 1 |

→ winner is **Tit for Tat** TFT

① start with C
② copy last move of opponent
→ • freundlich → don't start with D (they do better) , • forgiving → if other is freundlich, very high payoff
• non-exploitable → you get what you give ⇒ easy

**DOWNING**
→ estimate ex prob of cooperation $p_1(C|C_{\text{downing}})$ $p_2(C|D_{\text{downing}})$

$p_1 < 0{,}5 \rightarrow D$ (non coop)
$p_2 > 0{,}5 \rightarrow D$ (sfruttabile)
else → C
start with D

**JOSS** → like TFT, but 10% D after other C

**TFZT** Tit for two tat (could have won) → persona di più
→ less robust than TFT

## ECOLOGIC TOURNAMENT
(select/evolution without mutations)
→ the more you win the more points, the more you can "reproduce"

↱ is NG

**TFT:** doesn't win single match → but average
(bad guys win against exploitable, but lose to themselves)
↳ not SPE (subgame perfect equilibrium) → see error

**unendlich-game:** → not TFT, but its warranty as winner of PD

NG: TFT  $s(TFT, TFT)$ is NG
→ who selects D is punished
→ other strategies "freundlich" give same payoff → non strict NG
$u(s(TFT,TFT)) = u(s(TF2T, TFT))$
$s(\bar{C},\bar{C})$ not NG
$s(\bar{D},\bar{D})$ is NG

$T+P < 2R$

**TFTD:** like TFT, but starts D

$\bar{D}$ is not strict NG → $s(\bar{D}, TFTD)$ behaves exactly like $s(\bar{D},\bar{D})$

**es:** reciprocity help
Sioux & relationships
pipistrelli → food sharing: reciprocal help predicts durations more than relatedness

**RANDOM**
C/D with p=0,5

(weakness)
TFT/RAND: $y = \frac{S+3+1+0}{4} = 2{,}25$
$\bar{D}$/RAND $y = \frac{S+1}{2} = 3$

if TFT has error, it propagates (treadling hand)
CCCCDCDC...
CCCCCDCD...  TF2T would solve the problem

**NOVAK-SIGMOND:** PAVLOV  with ε-error
win → stay  (R,T)
lose → shift  (S,P)
↳ robust: even with error, after some time it cooperates again

**WW1** → soldier coop  chemical warfare
→ XMAS music
→ "we do the same to the opposite side" ⇒ let's cooperate
→ we give back 3 times as much

## B. WIEDERHOLTE SPIELE & SCHATTEN DER ZUKUNFT/VERGANGENHEIT (REPUTATION)

**THEOREMS:**
- in supergame, $\bar{D}$ is NG
- ∄ unconditioned coop strat that is NG
- ∃ conditioned coop strat NG if "discount parameter" is big enough

**diskontparameter** $w < 1$ : zukunft zählt weniger als present

$y_n = (1+r)^n y_0 \rightarrow y_0 = \left(\frac{1}{1+r}\right)^n y_n$

r: interesse  diskontparam, $w = 1/1+r \in [0,1]$

payoff $= v_0 + wv_1 + w^2 v_2 + \dots$

## TTTR (take money & run)

$\bar{D}$

$u(TTTR) = T + wP + w^2P + \dots + w^nP \rightarrow u(TTTR) - wu(TTTR) = T - wT + wP - w^{n+1}P$

$\Rightarrow u(TTTR) = \frac{T(1-w)}{1-w} + \frac{wP - w^{n+1}P}{1-w}$

$\lim_{n\to\infty} u(TTTR) = T + \left(\frac{w}{1-w}\right)P$

$u(TFT) = R + wR + w^2R + \dots + w^nR$  $\lim_{n\to\infty} u(TFT) = \left(\frac{1}{1-w}\right)R$

$u(TFT) \overset{!}{\geq} u(TTTR) \Rightarrow w \geq w^* = \frac{T-R}{T-P}$  ≡ Axelrods $\frac{1}{2}$

**w:** big means a lot of interaction later → cooperation evolution
low means less coop

FRIEDMAN is NG if $w \geq w^*$

**switchstrategy** $s$ = DCDC...

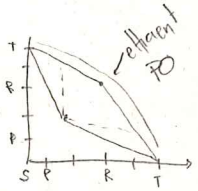
$\lim_{n\to\infty} u(s, TFT) = \lim \left(T + wS + w^2T + w^3S + \dots + w^{2n}T + w^{2n+1}S\right) = \frac{T + wS}{1-w^2}$

not better than simple cooperation, if $\frac{T+wS}{1-w^2} < \frac{R}{1-w}$

$w \geq v^* = \frac{T-R}{R-S}$

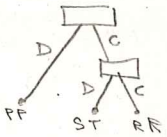
TFT is NG $\Leftrightarrow w \geq \max(w^*, v^*)$  Axelrod ≡ $\max(\frac{1}{2}, \frac{2}{3}) = \frac{2}{3}$

**folk theorem**
mixed strat: $p_1$ C: p D: 1-p
$p_2$ C: q D: 1-q




deviazione: triggers start with only D
→ strategies are NG, if zukunft enough strong is (no TTTR) → coop with threat

**options:**
- rules in tournament (mutations)
- territorio
- C,D, exit
- group coop




**other types:** VERTRAUENSPIEL (investing GD)

**Reputation**
→ not only self transactions/interactions known
→ also with other players

good rep $R^+$ if they never played D (seller) / never coop with $R^-$, only $R^+$ (buyer)  $p_2$ $p_1$
then $p_1$: $(C|R^+, D|R^-)$, $p_2$: C  is NG
incentive a cooperare anche solo per 1 transaz

C(repeated player, partner) > C(reputation) > C(stranger, 1 play)

why give **feedback?** (small cost) → collective good
but neoeconomics doesn't rate → system collapses

**eBay:** two-way rating system
(you get more if you give more → reciprocity)
→ illegal markets

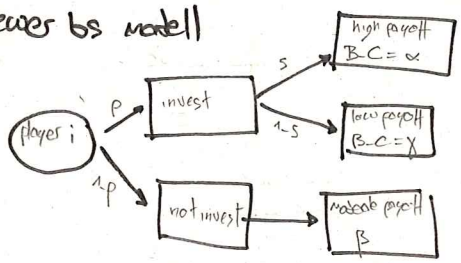
# 9. WETTBEWERB, ALTRUISMUS, RECIPROCITY

## Tocqueville paradox

→ a repressive regime runs greater risks of being overthrown when it tries to reform itself
→ + search in hope & reform

## Bourdons wettbewerbsmodell

N actors
k positions
n investors
s prob of winning
p prob to invest




$$s = \begin{cases} \frac{k}{n} & k < n \\ 1 & k \geq n \end{cases}$$

$$f(k,n) = \begin{cases} \frac{k}{n}\alpha + \frac{n-k}{n}\gamma & k < n \\ \alpha & k \geq n \end{cases}$$

$\alpha > \beta > \gamma$

| f(k,1) | f(k,2) ... |
|---|---|
| β | β |

## Beautiful mind game (not really NG)

N actors compete for scarce resource. only 1 will be successful
here k=1, γ=0

| compete / invest → | 0 | 1 | 2 | ... | n-1 ← other competitors |
|---|---|---|---|---|---|
| D | v | v/2 | v/3 | ... | v/n | dumb |
| C | β | β | β | ... | β | smart |

## mixed NG in competition game ⚠ only ES!!

$$f(inv) = (1-p)^{N-1} \cdot f(inv, n-1=0) + \binom{N-1}{1}(1-p)^{N-2} p \cdot f(inv, n-1=1) + \binom{N-1}{2}(1-p)^{N-3} p^2 f(inv, n-1=2) + \dots + p^{N-1} f(inv, n-1=N-1) \overset{!}{=} \beta = f(\neg inv)$$

ES N=6, k=1, α=9, γ=5, β=6,5

| p \ #of other investors | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| p | 9 | 7 | 6,3 | 6 | 5,8 | 5,7 |
| 1-p | 6,5 | 6,5 | 6,5 | 6,5 | 6,5 | 6,5 |

k=1 → winner takes it all → $p^* = 0{,}429$

## Altruismus vs. two economics

| Altruismus | two economics |
|---|---|
| rational | rational |
| interested in higher payoff for others (even if it costs me) | interested in own profit, material |

## behavioural GT

(altruism, reciprocity, social norms, trust, reputation, status, signal, conflict, coop, social order)

ES. Sellhorn → mostly honest

## Ungleichheitaversion (Fehr & Schmidt)

$u_i = f(\$) - g(\text{deviation from fairness reference})$

für N=2 $\quad u_i(x_i) = x_i - \alpha_i \max(x_j - x_i, 0) - \beta_i \max(x_i - x_j, 0)$

$\alpha_i, \beta_i$: personparameter
$u_i(x_i)$ = use of $ $x_i$ to person i
$\beta_i \in [0, 1[ \quad (\leq \alpha_i)$
↳ means lost by unfairness loss by paying

ES. PD $\quad u(T) = T - \beta(T-S)$ $\quad u(R) = R$ $\quad$ if $u(R) \geq u(T)$ we have NG (coop)
→ required $\beta \geq (T-R)/(T-S)$

# Dictator & ultimatum game

(dictator game → proposer, other accepts or rejects)

naive scenario

every subdivision is PO NG!

only (1,0), (1-ε, ε) with ε smallest unit are teilspielperfekt

fairness in dictator (% d) & ultimatum (% u)

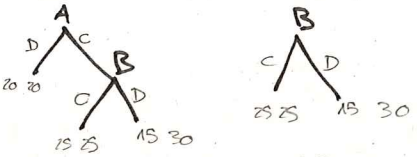
% d > % u
% d = % u
% d < % u

ES: fairness in stores
(give up to 50%)
but give more in ultimatum

## experiments:

→ no society represents naive scenarios
→ but variation from 50%
→ coop depends on economic wisdom
→ groups in classroom
→ altruism find a or sets punish

## Reciprocity:




65% B → C (reciprocity)
33% B → C

almost all cultures show this trait
→ esecuzione punishment / social norms / incentive coop

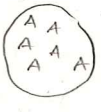
variations → accept less change if they know partner will get better deal

# 10. EVOLUTIONARY GAME THEORY

## fitness

→ the success of a strategy depends on frequency of others
→ same strategies related with genes
↳ mutations/variations depending on fitness (children)
learn/mistake → propagation
error → mutation

## ESS → evolution stabil strategien




STABIL: if $f(A,A) > f(J,A)$ ∀ mutations J → A is ESS

ES L/R fahrer start A = L (could also be R)

| | R | L |
|---|---|---|
| R | M | oo |
| L | oo | M |

subsubtitle unknown non antiana papahana (also every mixed s.)

## Falke - Taube

| | F | T |
|---|---|---|
| F | (V-W)/2 (V-W)/2 | V, 0 |
| T | 0, V | V/2 - t V/2 - t |

f: attack
T: nothing
symmetric
↳ costs of stor conflict

V > W → α: V-W/2 | V ; 1-α: 0 | V/2 - t

$E(F,F) > E(J,F) = \alpha E(FF) + (1-\alpha) E(T,F)$

only falcons

V < W → CG (only fak → dub strategy & vice versa)
pure strategy not ESS

V reverse
W more fento

→ which mixed start is stabl, ESS

## ESS (Maynard-Smith & Price)

→ small amount mutations

→ strategy I is ESS ⇔ • $E(I,I) > E(J,I)$

or • $E(I,I) = E(J,I) \wedge E(I,J) > E(J,J)$

$E(I,J) \geq E(J,I)$

$E(\alpha^*, \alpha^*) = E(\alpha, \alpha^*) \wedge E(\alpha^*, \alpha) > E(\alpha, \alpha)$

→ indifferenz th $\alpha(V-W)/2 + (1-\alpha)V = (1-\alpha)(V/2-t)$

$\alpha^* = \frac{V/2+t}{W/2+t}$ $V\uparrow$ $W\downarrow$ (wound cost, less fatal)

→ see p69 for math

**ESS max** ess doesn't give max payoff

$\alpha_{max} < \alpha^*$ (defensive strategy)

**TFT is not ESS** { C b super, che è superb a sua volta da D

**Abnutzungskampf** (war of attrition)

→ conflict for V → who waits more receives it

→ max waiting time for player

$P_1: w_1$, $P_2: w_2$ $w_1 > w_2 \rightarrow P_1: V - w_2$, $P_2: -w_2$

→ no NG in pure

but mixed strat → ESS

$p^*(t) = \left(\frac{1}{V}\right) \exp(-t/V)$

operator verteilung

$E = \int_0^m (V - x) p(x)\,dx - m \int_m^\infty p(x)\,dx$

(player wins / opponent wins)

**Flies** wait for reproduction
success not dependent on wait-time → no incentive to change strat.

**tumor cells**

→ see p71

## Beauty contest game ⚠

→ N actors pick # in $[a, b]$ → goal: closest to $pn$, $p \geq 0$.
$n$ stochastic distribution

es $p = \frac{2}{3}$ $[a,b] = [0, 100]$
$n$ = mean

should be rational if others are irrational? → rational = 0 (NG)

**pratica:** media 22,8
→ winner: 14,7

↳ could use 3rd, 4th, ... degree of backward induction

## Level k thinking (bounded rationality thinking) ⑤

→ player chooses according to level of thinking k:

$z = 50 p^k = 50 \cdot \left(\frac{2}{3}\right)^k$

k=0 → player select # random
k=1 → best answer for k=0
k=2 → best answer for k=1

k=3 → z = 14,81 (people think on 2nd lvl)

→ converges against NG, 0

**rounds** the more rounds you play, the more the average ↓

**Not rational:** why don't people do rational:
→ compensione situazione
→ interpretazione errori
→ super che altri non sono razionali

**CKR (common knowledge rationality)**
→ bot convergence towards NG (nobody wins with it)

## M. UNVOLLSTÄNDIGE INFORMATION, SIGNALE, KOOPERATION

→ diploma come indicatore di preparazione, competenza

**UNVOLLSTÄNDIGE INFO** über produktivität di candidati

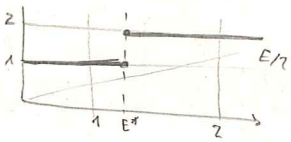
Model: group 1: productivity = 1, percent = $\alpha$
2: = 2 = $1-\alpha$

Pay the average: $\alpha \cdot 1 + (1-\alpha) \cdot 2 = 2 - \alpha$
but: goes towards 1 as high workers default

**bildung:** cost for $t$ years $g1$: $t$, $g2$: $t/2$

$2 - t^* < 1$, $2 - \frac{t^*}{2} > 1$ } $t^* \in ]1,2[$ ⇒ g1: not invested in ausbildung
g2: yes (t* years)

employer pays $w_1 = 1$, $w_2 = 2$




→ here it doesn't increase productivity, just for separating groups

## SIGNALS

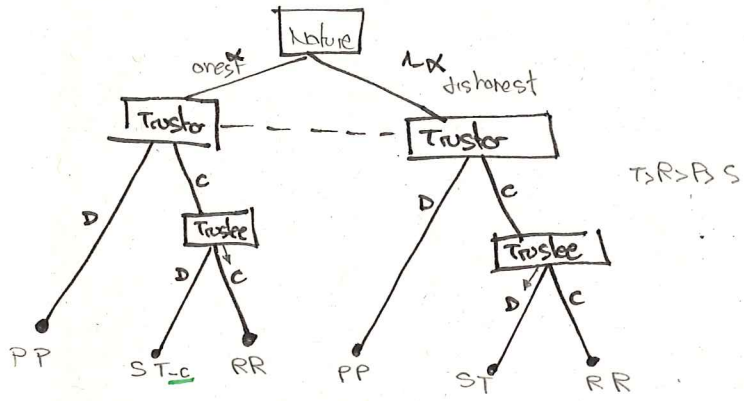
pavone → perché segnale se costo? segnale ti salute qualità

**handicap-principle Zahavis** (costi fanno capire tuo valore, "te la puoi permettere")
signal for quality is trustworthy if it costs

**appartenenza a un gruppo**
gift giving, gift wrapping (for partner)
→ signal must be observable

# Vertrauensspiel unvollständige Info

Treugeber : investiert x
Treuhand : doubled → ehrlich / unehrlich
α (1−α) → würde ich investieren if α = 10, 30, 50 %?




T > R > P > S

C: $E = \underbrace{\alpha \cdot R + (1-\alpha)S}_{C} \overset{!}{\geq} P$

$\left[ \alpha > \alpha^* = \frac{P-S}{R-S} \right.$
trust if $\alpha > \alpha^*$
defect otherwise

**common knowledge** → every player knows its preferences (private info) but not mitspielers
know distribution of probability

## SIGNALING THEORY

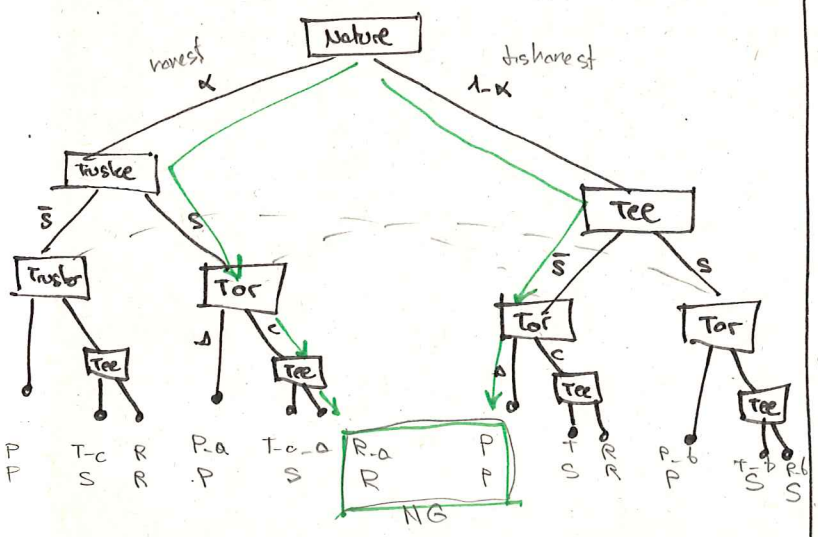
for receiver, signal from sender is observable thing ~~but~~ associated with unobservable trait / property
it has costs for the sender (decide if do it or not)
→ can distinguish type by type / frequency of signal

SENDER: signal must be worth it → receive higher payoff less cost for TRUE holder of property (type A) || higher payoff than B for signal type

RECEIVER: obtains advantage from noticing signal ⊙ gewünschte Lücke / cost difference between TYPES

## FULL GAME

with signals of trustworthiness




**signal cost:** $\bar{S} = 0$
$S = a$ (honest)
$b$ (dishonest)

c = cost of dishonest for honest
- R > T−c (sinnvoll)
- P < R−a (keep title consignale)
- P > T−b (stellen inutile consigne)

NG: ehrlich signals → Tor C
Tee dishonest doesn't → Tor D

If Typ B can fake signal with low costs → NG is broken

**Altruism** → costs tanto some signals
→ augments prestige (reward if society plays altruist)

os - birds that compete for altruistic behaviour

slides to check
- 5. Hollsumsp (+9)
- → 7b. Vorlesung
- → 8b. Verhandlung

# GT FORMULÆ

## Mixed strat

| | q | 1-q |
|---|---|---|
| p | a | b |
| 1-p | c | d |

$$p^* = \frac{d-c}{(a+d)-(b+c)}$$

$$q^* = \frac{d-b}{(a+d)-(b+c)}$$

## Dupol

$$\Pi_i = P(q_1+q_2)\cdot q_i - C_i(q_i)$$

$D(x) > C(x+1)$ } Social dilemma
$D(0) < C(N)$ }

## VoD:

$$E_i = (U-K)(1-q_i) + U\cdot q_i \cdot \left(\prod_{j\neq i}^{N} q_j\right)$$

$$\frac{\partial E_i}{\partial q_i} = K - U\prod_{j\neq i}^{N} q_i \doteq 0 \rightarrow \boxed{q_0 = \sqrt[N-1]{\frac{K}{U}}}$$

| | | $1-\Pi q_{j\neq i}$ | $\Pi q_{j\neq i}$ |
|---|---|---|---|
| | | C | D |
| p | $1-q_i$ C | U-K | U-K |
| | $q_i$ D | U | 0 |

Strong player:

$$p_i^* = 1 - \frac{U_i}{K_i}\left(\prod_{j=1}^{N}\frac{K_j}{U_j}\right)^{\frac{1}{N-1}}$$

$$P(\min 1\ \text{coop}) = 1 - q^N = 1 - \left(\frac{K}{U}\right)^{\frac{N}{N-1}}$$

## Axelrod

| | |
|---|---|
| 33 | 05 |
| 50 | 11 |

## Shatten zukunft

$$q_0 = (1+r)^n q_n$$

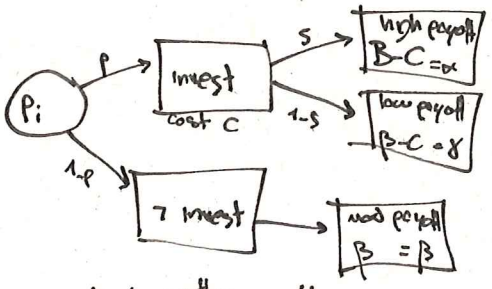
$$q_n = \left(\frac{1}{1+r}\right)^n q_0 = \omega^n q_0 \quad \omega \in [0,1]$$

$$\sum_{i=1}^{n} x^i = \frac{x^{n+1}-x}{x-1}$$

$$v(TFT) \geq v(\tau\, \Pi R) \Leftrightarrow \omega \geq \omega^* = \frac{T-R}{T-P}$$
$$v(S, TFT) \Leftrightarrow \omega \geq v^* = \frac{T-R}{R-S}$$

$$TFT \text{ is NG} \Leftrightarrow \omega \geq \max(v^*, \omega^*)$$

## WETTBEWERB




$\alpha > \beta > \gamma$

$$s = \begin{cases} \frac{k}{n} & k<n \\ 1 & \end{cases}$$

| | 0 | 1 | | N-1 ← other competitors |
|---|---|---|---|---|
| C | β | β | - - | β |
| D | E(k,1) | E(k,2) | | |

→ see computation

$$E(k,n) = \begin{cases} \frac{k}{n}\alpha + \frac{n-k}{n}\gamma & k<n \\ \alpha & \end{cases}$$

k positions
n investors

## UNGLEICHHEITAVERSION

$$U_i(x_i) = x_i - \underbrace{\alpha_i \cdot \max(x_j - x_i, 0)}_{\text{invidia}} - \underbrace{\beta_i \cdot \max(x_i - x_j, 0)}_{\text{gwshzia}}$$

## ESS

strategy I is ESS $\Leftrightarrow$ $\cdot E(II) > E(JI)$

or $\cdot E(II) = E(JI) \wedge E(IJ) > E(JJ)$

## Falke-Taube

| | F | T |
|---|---|---|
| F | $(V-w)/2$ <br> $(V-w)/2$ | V 0 |
| T | 0 V | $V/2 - t$ <br> $V/2 - t$ |

$E(FF) > E(JF) = \alpha E(FF) + (1-\alpha) E(FT)$

## Abnutzungskampf

(chi muore di più vince)

$$p^*(t) = \frac{1}{V} e^{-t/V}$$

$$E = \underbrace{\int_0^m (V-x)\, p(x)\, dt}_{\text{player wins}} - \underbrace{m \int_m^\infty p(x)\, dt}_{\text{opponent wins}}$$

## Level k thinking

$$z = 50 \cdot \left(\frac{2}{3}\right)^k$$

## Bildung

$z \cdot E^* < 1$

$z \cdot \frac{E^*}{2} > 1$

## Vertrauensspiel

$$E = \underbrace{\alpha R + (1-\alpha) S}_{C} \overset{!}{>} \underbrace{P}_{D} \rightarrow \alpha > \alpha^* = \frac{P-S}{R-S}$$

$R > T$ c

$P < R$ a

$P > T$ b

---

CG

| | C | D |
|---|---|---|
| C | 33 | 24 |
| D | 42 | 11 |

PD

| | C | D |
|---|---|---|
| C | 33 | 14 |
| D | 41 | 22 |

BOS

| ♂ \ ♀ | K | D |
|---|---|---|
| K | 43 | 22 |
| F | 11 | 34 |

SH

| | C | D |
|---|---|---|
| C | 44 | 13 |
| D | 31 | 22 |